B.Sc Part II (Physics Hons)
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Q (a) Obtain the necessary conditions that hold at the Surface of seperation between the two different dielectrics
(b) Prove that the law of refraction of lines of force at the surface seperating two dielectric.

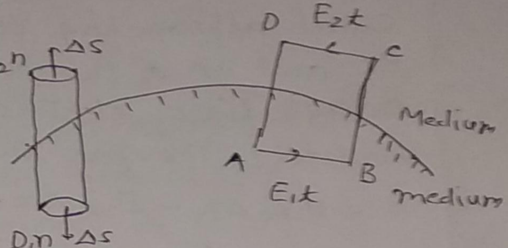
Ans (a) Boudary condition at the surface seperating two dielectric :→

The Change in electric field that occurs in going from medium to another is determined by the two basic ideas in electrostatics namely

(I) The first is Gauss's law i.e $\oint \vec{D}\cdot d\vec{s} = Q$, and

(II) The second is that an electrostatics field is a Conservative field, in other Words, no work is done in transporting a charge around a closed path in an electrostatic field i.e $\oint \vec{E}\cdot d\vec{l} = 0$

Let us apply Gauss's law to the cylindrical Surface of height h and base are ΔS. The cylindrical box is so Constructed that it lies half in each medium.



Let D_{1n} be the average normal component of displacement vector $\vec{D}$ to the bottom of the box in medium 1 and D_{2n}, the average normal component of Displacement vector $\vec{D}$ to the face of the box in medium 2. ~~D_{1n} is~~ D_{1n} is an inward normal. By making the height of the cylinder 'h' approaching zero, the Contribution of the curved Surface to the box is taken as zero. By Gauss's law the total flux $D_{1n}\Delta S - D_{2n}\Delta S = 0$ ——— (I)

where Q is the total charge enclosed by the surface.

The Second terms of L.H.S of this equation is negative because D_{1n} and ΔS are oppositely directed.

$$\therefore D_{2n} - D_{1n} = \frac{Q}{\Delta S} = \sigma \quad \text{- - - - - (2)}$$

where σ is the surface charge density on the boundary of the two media. According to eq^n (2) the normal component of the displacement vector $\vec{D}$ changes at a charged boundary between two dielectric by an amount equal to the surface charge density.

If the boundary is free from charge $\sigma = 0$, the from eq^n (2) $D_{2n} - D_{1n} = 0$

$$\therefore D_{2n} = D_{1n} \quad \text{- - - - - (3)}$$

Thus the normal component of displacement vector is continuous ~~as~~ across the charge free boundary between two dielectrics.

Since the electrostatic field is conservative the integral of $\vec{E}$ around a closed path

$$\int_{ABCD} \vec{E}\cdot d\vec{l} = \int_{AB} \vec{E}\cdot d\vec{l} + \int_{BC} \vec{E}\cdot d\vec{l} + \int_{CD} \vec{E}\cdot d\vec{l} + \int_{DA} \vec{E}\cdot d\vec{l} = 0$$

By making the path length BC is very small approaching zero, the work done along the segments BC and DA of the path normal to the boundary is even through finite electric field may exist normal to the boundary. Therefore the line integral of $\vec{E}$ around ABCD $\square$ is

$$E_{1t}\,\Delta x - E_{2t}\,\Delta x = 0, \quad \text{where } AB = CD = \Delta x$$

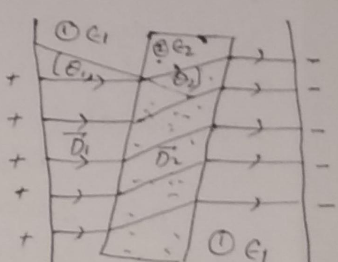
$$\therefore E_{1t} - E_{2t} = 0$$

$$E_{1t} = E_{2t} \quad \text{- - - - - - - - (4)}$$

Thus the ~~and~~ tangential components of the electric field are the same on both sides of a boundary between two dielectric. In other words, the tangential electric field is continuous across such a boundary.

<u>law of refraction of lines of the force of $\vec{E}$:→</u>

Let us consider a $\square$ infinite slab of material of permitivity ϵ_2 which is immersed in a dielectric medium of permitivity ϵ_1.



Let θ_1 be the angle which is normal to the boundary makes with the lines of electric ~~flux~~ force of medium 1 and θ_2 be that which the normal to the boundary makes with the lines of electric force in medium 2 (inside the slab).

Let $\vec{D}_1$ and $\vec{D}_2$ be the electric displacement vector outside and inside the slab respectively. and $\vec{E}_1$ & $\vec{E}_2$ be the corresponding electric field intensity vectors. Then according to eq^n^ (3) and (4)

$$D_1 \cos\theta_1 = D_2 \cos\theta_2 \quad \text{-----------} \quad (5)$$

& $$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{-----------} \quad (6)$$

(6)/(5) $$\frac{E_1 \sin\theta_1}{D_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{D_2 \cos\theta_2}$$

$$\therefore \frac{E_1}{D_1} \tan\theta_1 = \frac{E_2}{D_2} \tan\theta_2 \quad \text{-----------} \quad (7)$$

But for isotropic dielectric media

$$D_1 = \epsilon_1 E_1 \quad \text{and} \quad D_2 = \epsilon_2 E_2$$

from equation (7)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2} \quad \text{-----------} \quad (8)$$

This equation is called "Snell's law", in electrostatics. This gives the relation between the tangents of the angle of incident θ_1 and the angle of refraction θ_2 in terms of the permitivities of the two media.